

The METONIC CYCLE is the Moon's 19 year cycle where the Moon returns to exactly the same place (at the same longitude and against the same constellation) in the sky with the same phase.

An ordinary calendar is a solar (Sun) calendar. It keeps the dates in sync with the Sun. For instance, the Sun is at its highest point as we view it around the 21st of June - the summer solstice. The solar cycle (a year) takes 365 and a quarter days to complete. Every four years is a leap year when an extra day is added to the year. This accounts for the quarter day and keeps the date nearly in sync with the seasons. The cycle is not quite 365 and a quarter days. The error is 3 days in 400 years.

A lunar (Moon) calendar keeps the lunar dates in sync with the Moon. For example, the 1st of the month could be on the new Moon. Then the 8th of the month would fall at the waxing half Moon. The dates vary for different types of lunar calendars, Muslim, Chinese, Buddhist, Jewish etc. Some have the 1st on the full Moon. However, ALL follow the Metonic Cycle which keeps the lunar dates in sync with the Moon.

Tropical Lunar Month

The Moon returns to the same spot in the sky (against the backdrop of the same constellation) every 27.322 days which is called the Tropical Lunar month. However, the Moon's phase is not the same for two days.
Synodic Lunar Month.

The Moon returns to the same phase every 29.5306 days and is called the Synodic Lunar month. There are 12 synodic months and 13 tropical months (returns) in one year. See diagram showing how the Metonic Cycle is calculated:

Metonic Intervals	Synodic Month (SM)		Synodic Month (SM)	Tropical Month (TM)
I	12 synodic months	ends 11 days before	1 tropical year	13 tropical months (returns)
II	LLY 25 synodic months	ends 8 days after	2 tropical year	27 tropical months (returns)
III	37 synodic months	ends 3 days before	3 tropical year	40 tropical months (returns)
IV	49 synodic months	ends 14 days before	4 tropical year	53 tropical months (returns)
V	LLY 62 synodic months	ends 5 days after	5 tropical year	67 tropical months (returns)
VI	74 synodic months	ends 6 days before	6 tropical year	80 tropical months (returns)
VII	LLY 87 synodic months	ends 13 days after	7 tropical year	94 tropical months (returns)
VIII	99 synodic months	ends 2 days after	8 tropical year	107 tropical months (returns)
III+VIII=XI	37+99=	3 days before + 2 days after =	3+8=	40+107=
XI	136 synodic months	ends about 1+ day before	11 tropical years	147 tropical months (returns)
VIII+XI=XIX	99+136=	2 days after + 1+ days before	8+11=	107+147=
XIX	235 synodic months	SAME TIME	19 tropical years	254 tropical months (returns)

Therefore, it takes 19 years (or 6939 days) for the Moon to return to the same spot in the sky at same phase. This can be seen as:

19 tropical years - $365.24 \text{ days} \times 19 = 6939.56 \text{ days}$

235 synodic months - $29.5306 \text{ days} \times 235 = 6939.691 \text{ days}$

254 tropical months - $27.322 \text{ days} \times 254 = 6939.788 \text{ days}$

Lunar Leap Year (LLY)

To keep the Moon's cycles as close to the Sun's cycle, an extra synodic month and an extra tropical month are added. So instead of 12 synodic months and 13 tropical months in a year, it is 13 synodic months and 14 tropical months. $TY = TM - SM$

Tropical year = tropical month MINUS synodic month. To calculate large numbers of cycles, metonic intervals can be added (eg. III + VIII = XI)